MIE1605H: Stochastic Processes (Fall 2021)

- **Instructor:** Prof. Vahid Sarhangian, Office: BA8108, Email: sarhangian@mie.utoronto.ca
- **Office hours:** TBD in class.
- **Lectures:** Thursdays (9:00-12:00AM); First class: September 16th.

Course description and prerequisites

This course is an introduction to stochastic processes with an emphasis on applications to queueing theory and service Engineering. More specifically, the following topics will be covered:

- **Fundamental Methods from Probability Theory and Calculus**: Basic probability theory; Conditional probability and expectation; Bounded, finite, and infinite; Infinite series and limits; Convergence theorems.
- **Discrete-time Markov Chains**: Recurrence and transience; Communicating states and irreducibility; Stationary distributions; Markov chain convergence.
- **Martingales**: Stopping times; Optional stopping theorem; Wald's theorem; Martingale convergence theorem.
- **Brownian Motion:** Construction of Brownian Motion from a random walk; Diffusion processes.
- **Poisson Processes and Continuous-Time, Discrete-Space Markov Processes**. Definition; Construction; and examples.

The concepts and methods will be illustrated using various examples and applications, including:

- **Fundamental Probabilistic Models**: Random walks; Gambler's ruin; Ehrenfest Urn; Branching processes; Markov Chain Monte Carlo (MCMC) algorithms.
- **Financial Applications**: Geometric Brownian Motion; Option pricing in discrete and continuous time.
- **Queueing Networks and Approximations**: Birth-death queues; Jackson queueing networks; Fluid and diffusion approximations of queueing networks.

The course is designed for graduate research students and mathematical rigour is emphasized throughout the course. MEng students require permission from the instructor to enroll in the course (details to be announced in class). Students are expected to have a strong undergraduate (non-measure theoretic) level background in probability at the level of MIE231, STA347, or equivalent, and ideally have some knowledge of real analysis. It is recommended that the students take MIE1613 (Stochastic Simulation) before this course.

Textbook and additional references

Notes will be provided in class. The main reference for the course is:

- J. S. Rosenthal (2019) A first look at stochastic processes. World Scientific Publishing Company, Singapore,

which can be purchased from the publisher's website or Amazon.ca.

Some of the topics and applications will be based on the following texts, which are also useful sources for supplementary readings.

- R. Durrett (2012) Essentials of stochastic processes. Springer, New York.
- J. S. Rosenthal (2006), A first look at rigorous probability theory. World Scientific Publishing Company, Singapore.
- C. Hong, and D. Yao (2013) Fundamentals of queueing networks: Performance, asymptotics, and optimization. Springer Science & Business Media.

Durrett (2012) is available online on the publisher's website and here. Additional resources and papers will be posted on the course webpage on Quercus.

Evaluation

- 30% Homework (To be assigned after covering each major topic)
- 30% Midterm (Date and time to be announced)
- 40% Final test (Date and time to be announced)

Tentative Course Plan

- Introduction and probability review (1 lecture)
- Discrete Markov Chains (4-5 lectures)
- Random walks and Martingales (2 lectures)
- Brownian Motion and diffusion processes (1-2 lectures)
- Poisson Processes (1 lecture)
- Continuous-time Markov processes (2 lectures)
- Exact and approximate analysis of queueing networks (1-2 lectures)