MIE 1452: Signal Processing - Fall 2020

1/ Instructors:

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2/ Lecture Times: Wednesday, 1:00 – 3:00 pm (tentative)

3/ Lecture Location: TBA

MIE 1452: Course outline (contd.)

4/ Course Background:

The course is targeted towards senior graduate students who are collecting signals in an experimental research program. Signal Processing requires a lot of math! Particularly useful background knowledge:

- Good familiarity with complex numbers
- Some familiarity with Fourier series or Fourier transforms
- Familiarity with a mathematical package such as MATLAB
- Good comfort level with statistics

5/ Course Organization:

The course is divided into two main modules:

a) Analysis of systems (Lectures 1-6)

b) Random signals, Power Spectra, filtering and adaptive systems (Lectures 7-13)

MIE 1452: Course outline (contd.)

6/ Grading Scheme:

Assignments (Lectures 1-6) – 5%

Assignments (Lectures 7-12) – 5%

Midterm (closed book – formula sheet will be provided) – 20%

Term project – 30%

Final exam (closed book – formula sheet will be provided) – 40%

All relevant course materials will be posted on Quercus

Lecture Topics

Lecture No.	Date	Topics
1	Sept 16	Overview of systems (Biomedical/mechanical), key components, signal characteristics. Continuous and discrete-time signals. Frequency content. Linear time-invariant systems. Linear constant-coefficient differential equations delta functions. Impulse response. Signal convolution. Definition of Fourier Transform.
2	Sept 23	Fourier Transform. Signal convolution and de-convolution in time and frequency domains. Data acquisition systems. Phase plots. Periodic and aperiodic signals; discrete and continuous signals in the frequency domain. Examples of DFT, DTFT. Parseval's theorem. Signal sampling and windowing: distortions and strategies. Nyquist theorem. FFT. Optimization of signal acquisition parameters.
3	Sept 30	
4	Oct 7	
5	Oct 14	Signal Modulation. Selection of signal windowing kernel. Signal scalloping. Introduction to low-pass signal filters.
6	Oct 21	Noisy signals. auto-correlation and cross-correlation functions. signal chirps. analytic signal. Applications.
7a	Oct 28	Mid –Term (first half of lecture period). It will be based on all lectures delivered by Sinclair up to this date (1 hour)
7b	Oct 28	Term Projects

Lecture Topics (contd.)

Lecture No.	Date	Topics
8	Nov 4	Random noise, Mean, Variance and Moments, Uniform and Gaussian noise. Binary detection, Receiver operating Characteristics, Baysian classifier
9	Nov 11	Random processes, Stochastic processes, Stationarity, Ergodicity, Autocorrelation, Cross correlation. Frequency domain representation of Random Processes – Power Spectra, Periodogram, Parametric Models of PSD estimation. AR, MA and ARMA models.
10	Nov 18	
11	Nov 25	Z-transform, Digital Filters. FIR, IIR, Weiner filters, Adaptive filters, adaptive line enhancer. Examples fetal ECG.
12	Dec 2	
13	Dec 9	Kalman Filters, PCA, ICA
14	Dec 16	Final exam

General Reference Materials on Signal Processing

A.V. Oppenheim and A.S. Willsky, **Signals and Systems**, Prentice Hall, 2nd edition (1996). ISBN 0-13-814757-4
This text is available from Amazon, and there are a few copies in the engineering library. The first edition of this text is organized slightly differently, but has primarily the same material.

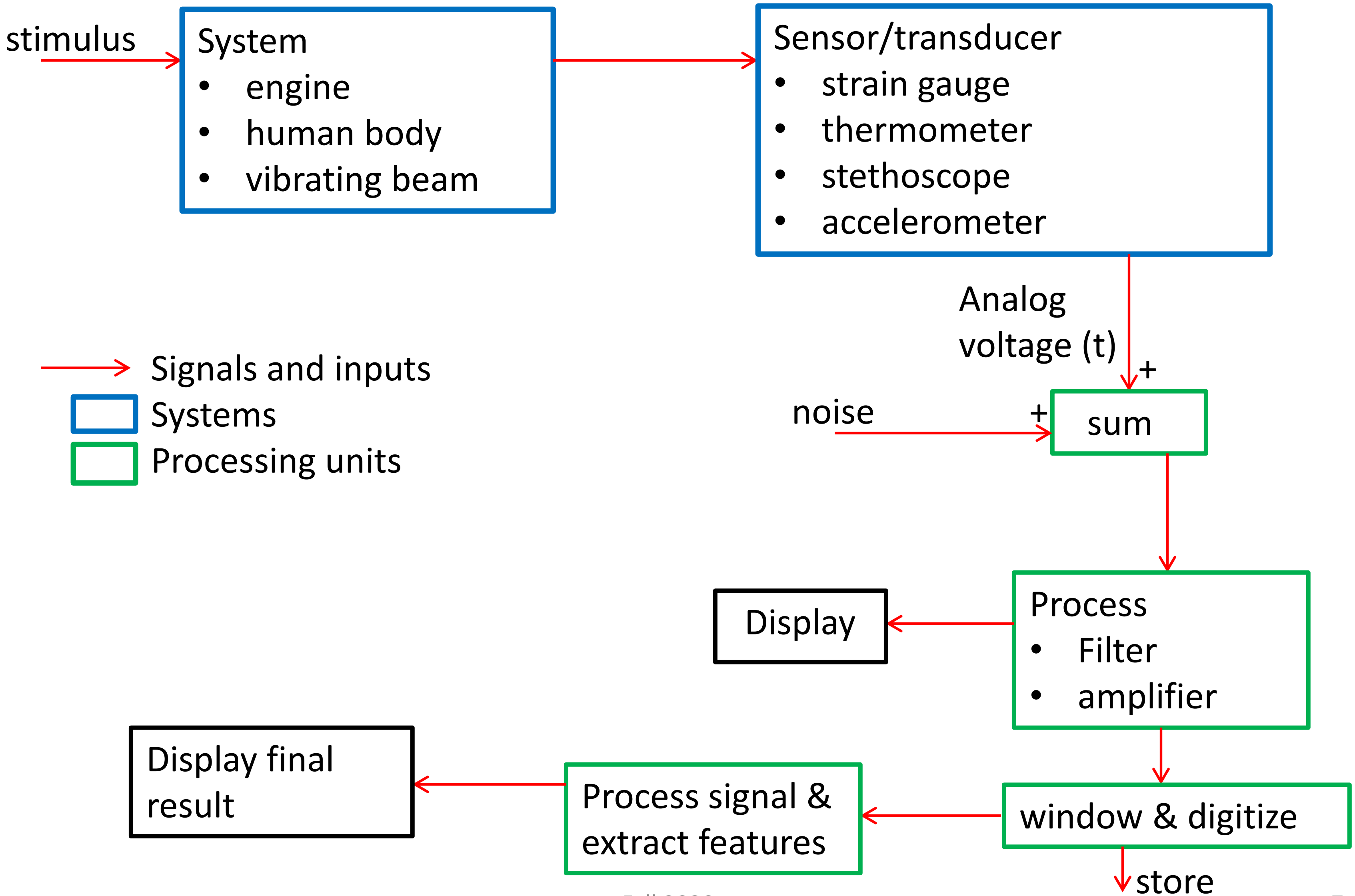
A. Papoulis, S.U. Pillai, Probability, **Random Variables and Stochastic Processes,** McGrawHill, 4th Edition.

Leif Sornmo and Pablo Laguna, **Bioelectrical Signal Processing in Cardiac and Neurological Applications,** Elseveir, Academic Press

M.H. Hayes, **Digital Signal Processing**, Schaum's outlines, McGraw Hill, 2nd edition (2012). ISBN 978-0-07-163509-7

H. Hsu, **Signals and Systems**, Schaum's outlines, McGraw Hill (2011)

Experimental Laboratory Program



Course objectives

- Ultimate course objective is *to characterize properties of main experimental system, by looking at its response to stimuli*.

  How? Look at the system's response to various stimuli.

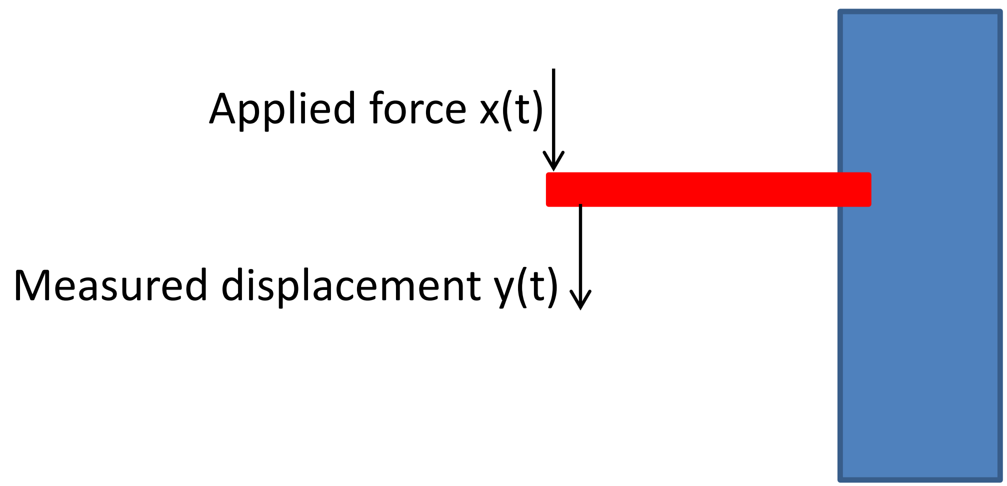
  … but the response is hard to interpret $\rightarrow$

- Our intermediate objective is to select appropriate test stimuli, then process the response signal from the system, to extract key features.
- We also look at various methods to display the system output to clarify the key points

First: Some basic properties of signals and systems (ref: Oppenheim)

Linear Time Invariant (LTI) system:

e.g., system is a clamped beam, x(t) is applied force; y(t) is displacement



Definition of linear:

If $x_1(t) \rightarrow y_1(t)$, and $x_2(t) \rightarrow y_2(t)$

Then: $c_1x_1(t) + c_2x_2(t) \rightarrow c_1y_1(t) + c_2y_2(t)$

Which real systems are truly linear??

LTI systems – some key ideas (contd.)

1/ Linear systems usually feature small inputs
(almost all systems go non-linear if $x(t)$ is large enough)

2/ Time invariance: Almost all systems have some time variance
(beam heats up, background noise changes …)

3/ Life is a lot simpler if a system has a unique, linear, time-invariant transfer function that links any input $x(t)$ to its output $y(t)$

4/ If we can figure out what that transfer function is, then we know everything that there is to know about our LTI system.

We will assume that our systems are LTI in this course

Analog vs Discrete-Time Signals

- An analog (continuous-time) signal y(t) is defined for all time t over a finite or infinite range. Very often, analog signals are in the form of a voltage v(t). It may have some jump discontinuities.

- A discrete-time signal y[n] is defined only at discrete points. It may be *naturally* discrete:

  - e.g., monthly rainfall in year 2012, for n = 1, 2, …12

  - e.g. diameters of the planets in our solar system, n = 1, 2, …9

- *Or* y[n] may be the result of sampling y(t) at discrete points in time. This will often be the case in an experiment.

Why digitize a signal?

- By sampling y(t), we can store the result digitally as a vector of length N, where n=1,2,3,4,…N. (Be on the lookout for cases where n = 0,1,2,…N, such that you have a vector of length N+1).
- Together with the N data values, you will also need the sampling interval Δt: $y[n] \Rightarrow y[n\Delta t]$
- Depending on the type of experiment, you may also be interested in the starting time t_o.
- Digitized signals can be readily stored on a computer, or manipulated to extract features of interest.
- A modern oscilloscope can digitize a signal. Oscilloscope specifications of interest are:
    - Number of channels
    - Bandwidth (Note: bandwidth is typically specified in terms of 3 dB drop-off)
    - Memory buffer size
    - Number of bits in each measurement
    - Data transfer rate to a computer
    - Computational power

e.g., Digitized (discrete) Cosine Signal

(a) y[n] = cos[nΔt] (in radians),
with Δt measured in seconds

(b) n = 0, 1, 2, … N-1

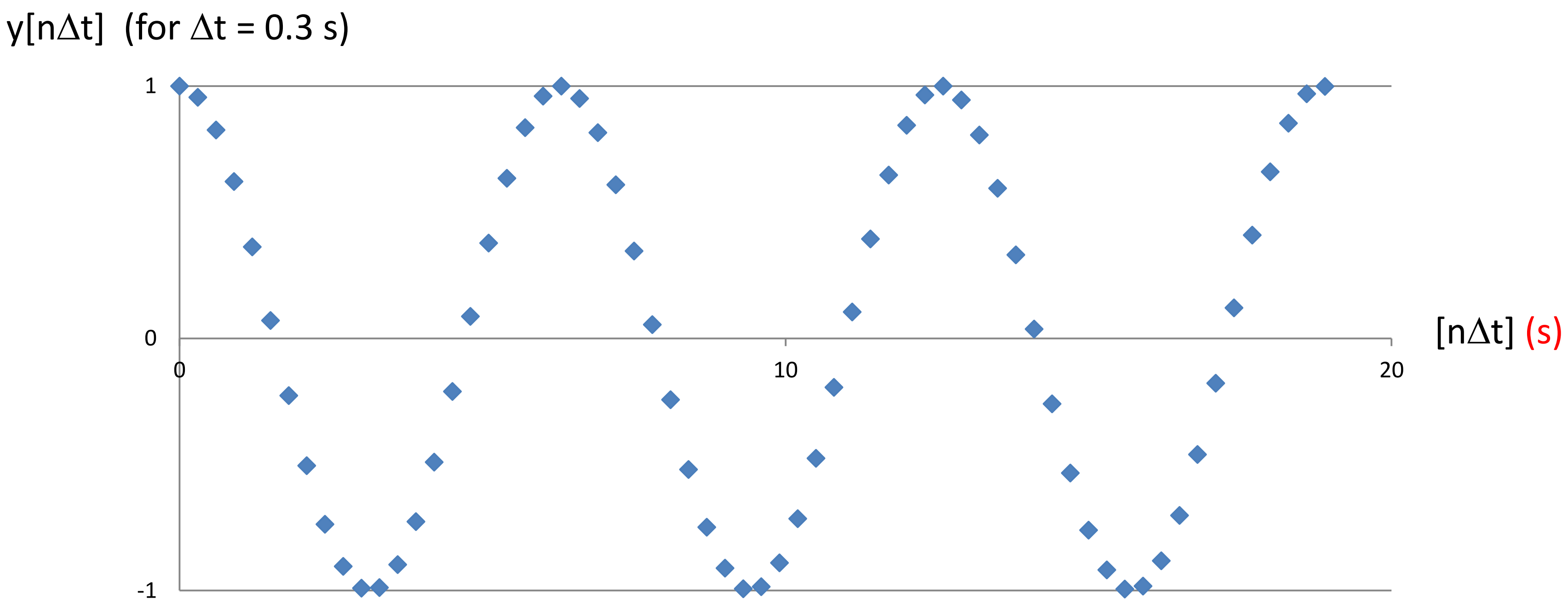
(c) N = 64 *(N or N+1 data points?)*

(d) n can have negative values

(e) y[n] or y[nΔt] defined at discrete points

(f) cos function takes radians as argument

(h) Δt = 0.3 seconds



Time invariant Process "B"

Suppose $x[n] \to y[n]$: $x[n] \to \boxed{B} \to y[n]$

Then "B" is time invariant iff: $x[n - n_o] \to y[n - n_o]$ for any sequence $x[n]$, any n_o

Similarly,

Suppose $x(t) \to y(t)$

Then "B" is time invariant iff: $x(t-t_o) \to y(t-t_o)$ for any $x(t)$, any t_o

(Often, *RLC* electrical circuit components are considered to be time invariant, although they actually change as they warm up to their normal operating temperature.)

Periodic signals

$x(t) = x(t + \tau)$, period = τ

(Note that this signal is also periodic with period 2τ, 3τ, 4τ, but we quote only the "fundamental" period τ.)

$x[n] = x[n + N]$, period N

$x[n\Delta t] = x[n\Delta t + N\Delta t]$, period $\tau = N\Delta t$ $\big($or $x[n] = x[n+N]$ $\big)$

e.g. $x(t) = \cos(5t) = \cos(5(t + 2\pi/5))$ $\Rightarrow$ periodic signal, $\tau = 2\pi/5$ (units?)

e.g. $x[n] = cos\left(2\pi\frac{n}{15}\right) = cos\left(2\pi\frac{n+15}{15}\right) \Rightarrow$ periodic signal, N = 15

How about $x[n] = \cos(n)$, n=0, 1, 2, …? Why is this *not* periodic?

Even and Odd Signals

Even signals: $x(t) = x(-t)$

e.g. $x(t) = 5\cos(t)$

$x(t) = |t|$

$x(t) = 3t^2 + 5t^4$

$x[n] = n^2$

$x[n] = |n(\Delta t)| + 7\cos(3\pi n(\Delta t))$

Odd signals: $x(t) = -x(-t)$

e.g. $x(t) = t + 3t^5$

$x(t) = 6\sin(5t)$

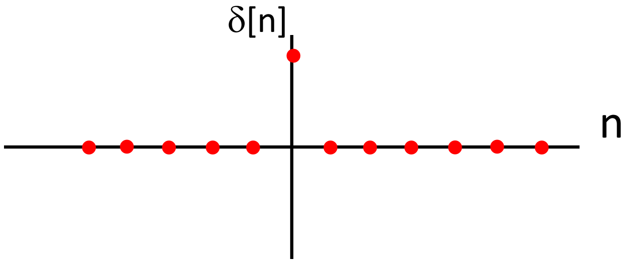
$x[n] = 4n^3 + \sin(3n)$

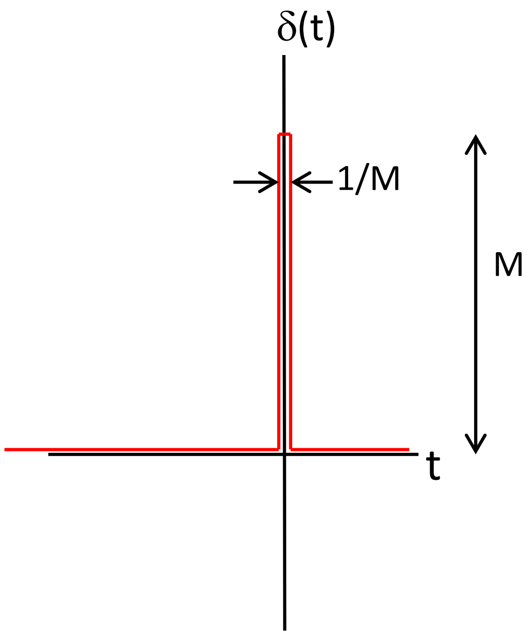
Note that most signals are neither even nor odd.

Can *any* real signal be broken up into a sum of even plus odd signals? (Consider Fourier's theorem)

Delta function, δ(t) or δ[n]

Discrete signals: $\delta[n] \quad = \quad \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$

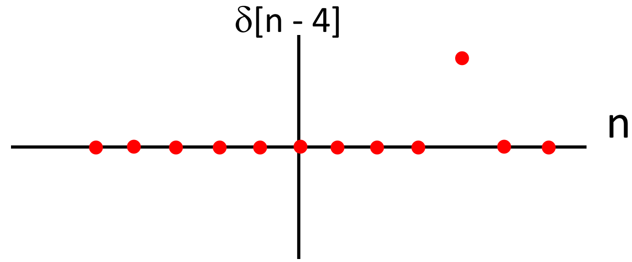




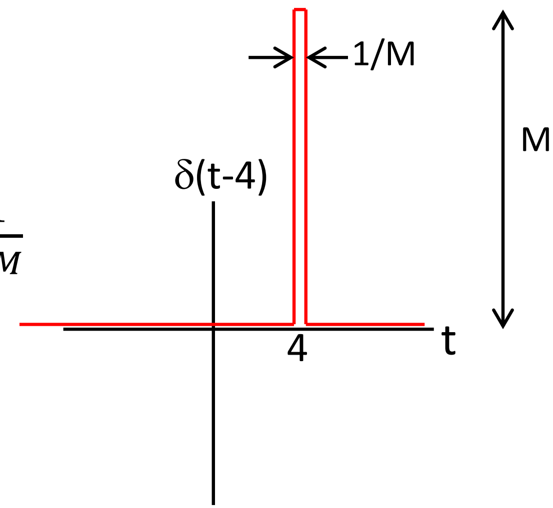
Analog signals: $\delta(t) \quad = \lim_{M\to\infty} \begin{cases} M, & -\frac{1}{2M} < t < \frac{1}{2M} \\ 0 & otherwise \end{cases}$

Shifted delta function, δ(t) or δ[n]

Discrete signals: $\delta[n-4] \;=\; \begin{cases} 1, & n = 4 \\ 0, & n \neq 4 \end{cases}$



Analog signals: $\delta(t-4) = \lim_{M \to \infty} \begin{cases} M, & 4 - \frac{1}{2M} < t < 4 + \frac{1}{2M} \\ 0 & otherwise \end{cases}$



Key properties of δ(t) – analog signals

1/ $\int_{-\infty}^{+\infty} \delta(t)dt \; = \; \int_{-1/2M}^{1/2M} M\, dt = M \left(\frac{1}{2M} - \frac{-1}{2M}\right) \; = \; \frac{M}{M} \; = \; 1$

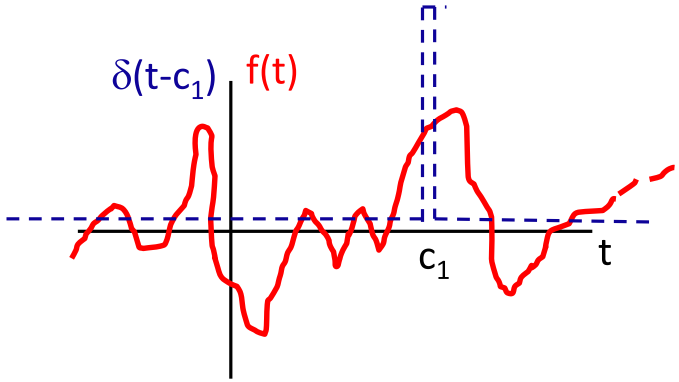
For any continuous function *f*(t), defined in the vicinity of t=0,

2/ $\int_{-\infty}^{+\infty} f(t)\delta(t)dt = \int_{-1/2M}^{1/2M} f(0)M\, dt = M\, f(0)\left(\frac{1}{2M} - \frac{-1}{2M}\right) \; =$

$$= f(0)\,\frac{M}{M} = \; f(0)$$

3/ $\int_{-\infty}^{+\infty} f(t)\delta(t - c_1)dt$

$$= \int_{-M/2}^{M/2} f(c_1)M\, dt = f(c_1)$$



Key properties of δ[n] – discrete signals

Analogous to the case of analog signals,

1/ $\sum_{n=-\infty}^{n=+\infty} \delta[n] = 1$

2/ $\sum_{n=-\infty}^{n=+\infty} x[n]\delta[n] = x[0]$

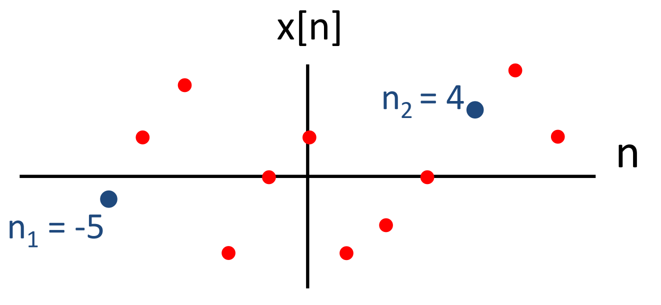
3/ $\sum_{n=-\infty}^{n=+\infty} x[n]\delta[n-7] = x[7]$

Signal Energy

- $E = \int_{t_1}^{t_2} |x(t)|^2 dt$, or $E = \sum_{n_1}^{n_2} |x[n]|^2$
- E_∞ refers to the limiting values as the time period extends to infinity
- Note that signal energy is also defined for complex signals
- Note that this definition is proportional to our conventional engineering definition of energy, but may be missing a multiplication constant. E.g.,

  energy consumed in a resistor R is $E = \int_{t_1}^{t_2} R|i(t)|^2 dt$

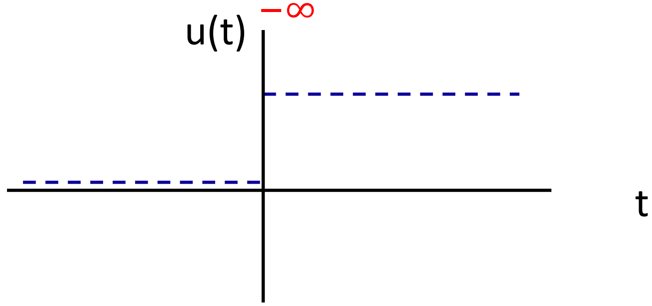
- Power = Energy/time
- Be careful of discrete indexing: Power $= \frac{1}{n_2 - n_1 + 1} \sum_{n_1}^{n_2} |x[n]|^2$



Unit Step Function = Heaviside Function

- u(t) = $\begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$ (undefined at t = 0, or sometimes defined as u(t) = 1 for t =0)
- u[n] = $\begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$
- From the definitions of $\delta(t)$ and *u(t),*

$$u(t) = \int_{-\infty}^{t} \delta(s)ds \qquad or \qquad \delta(t) = \frac{du(t)}{dt}$$



Derivative of u(t) is zero everywhere, except at t=0 where the derivative of u(t) becomes infinitely large.

Impulse response h(t)

- Consider the case where we input an impulse $\delta(t)$ to a LTI system. The resulting output is defined as the impulse response $h(t)$ of the system:

$$\delta(t) \longrightarrow \boxed{\text{LTI system}} \xrightarrow{\quad h(t) \quad}$$

- A real system must be causal, meaning that it can not react to an input signal until the input signal has actually happened. (That is, it cannot anticipate what the future input signal will be.)

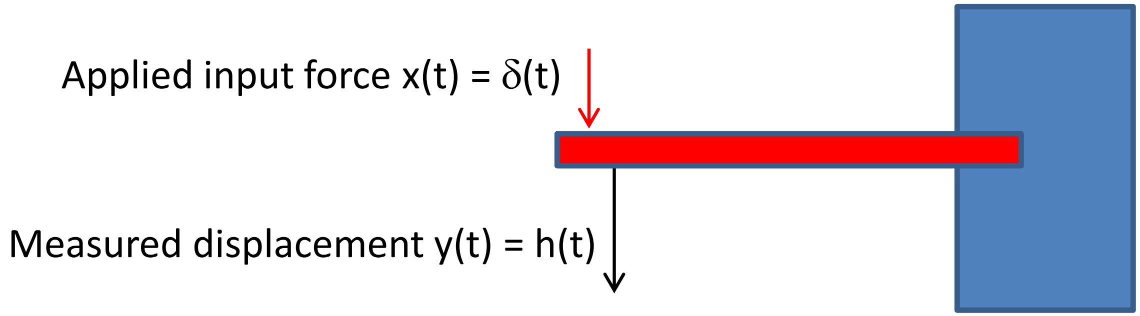
- Therefore, for a causal system, $h(t) = 0$ for $t<0$

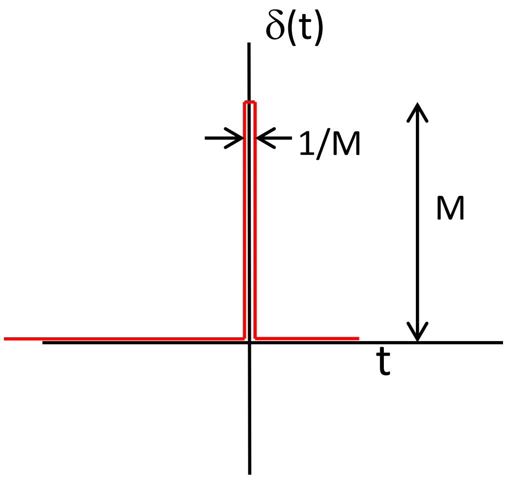
- E.g. What is the impulse response of a perfect amplifier that multiplies any input signal by a factor of 5?

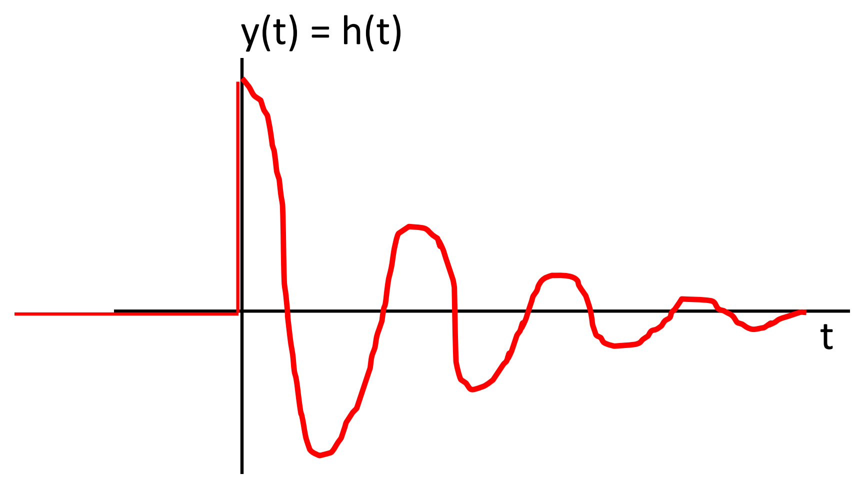
  Answer: If the input is $\delta(t)$, then the output will be $5\delta(t)$, so

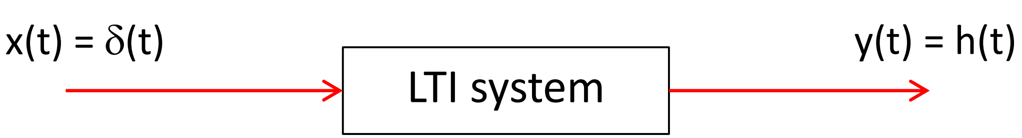
  by definition, impulse response $h(t) = 5\delta(t)$

More advanced impulse response example



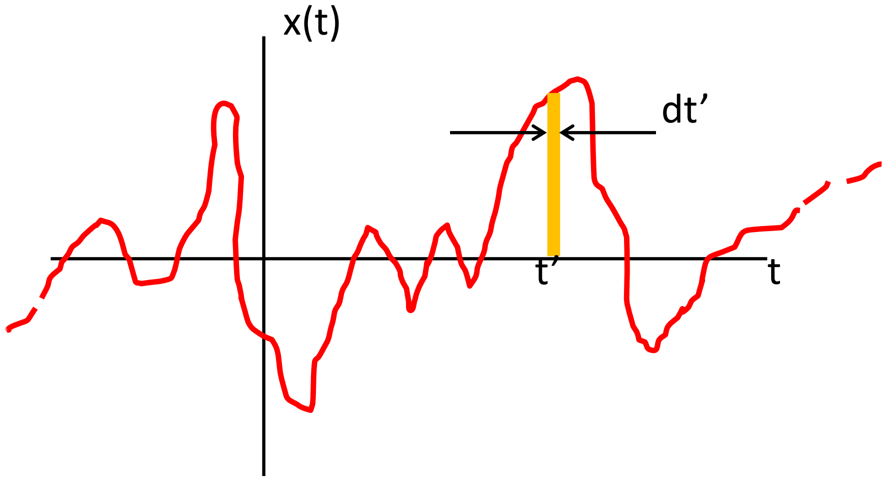


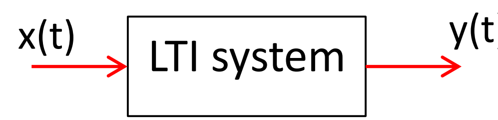




Important theorem for LTI systems

Suppose we know the impulse response h(t) for a LTI system. What would be the system response y(t) for *any* arbitrary input x(t)?





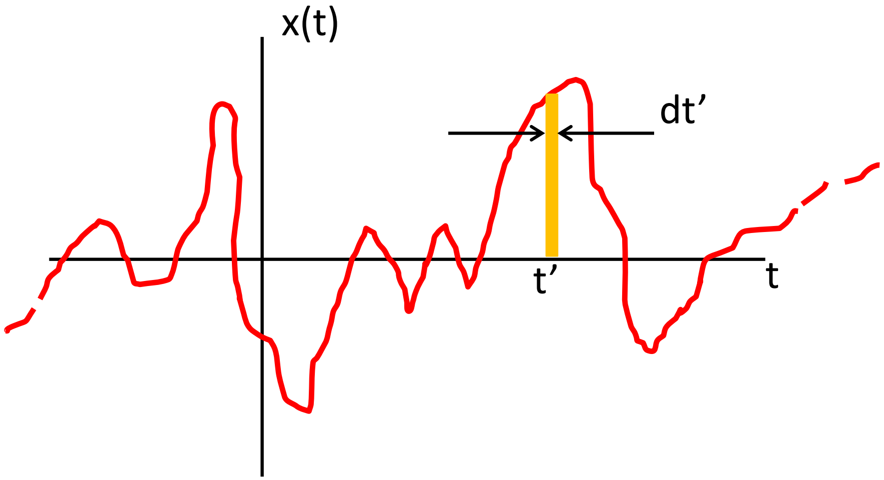
Answer: We can break up the function x(t) into a bunch of extremely narrow rectangles. One of these rectangles, centered at time t = t', is shown.

Note: The orange rectangle is very similar to a delta function, with the following two important differences:

1) The rectangle is centered at t = t', while δ(t) is centered at t=0.

2) The rectangle has an area = x(t') dt' , while δ(t) has an area equal to 1.

Important theorem for LTI systems (contd.)



Output y(t) due to orange rectangle input:

Note 1: If orange rectangle were a delta function, then y(t) would be h(t)
Note 2: If orange rectangle were a delta function time shifted to t = t', then y(t) would be equal to h(t – t')
Note 3: Orange rectangle is actually a delta function time shifted to t=t', and scaled down to an area of only x(t') dt', so y(t) = (x(t') dt') h(t – t')

To get total output, sum the contributions from all of the orange rectangles by integrating:

$$y(t) = \int_{-\infty}^{+\infty} x(t')\, h(t - t')\, dt'$$

Important theorem for LTI systems (contd.)

If we know h(t) for a LTI system, then we know everything about the system. Specifically, we can calculate y(t) for *any* input x(t), if we know the impulse response of the system.

Definition: $\int_{-\infty}^{+\infty} x(t')\, h(t - t')\, dt'$ is called a *convolution integral* = *x(t) * h(t)*

Note 1: Convolution integral is symmetric: x(t) * h(t) = h(t) * x(t)

Note 2: The "minus" sign in the convolution integral is important. Don't omit it!

Note 3: Calculating a convolution integral numerically for a large range of *t* values is computationally expensive

Note 4: Recall that h(t) = 0 for t < 0 if the system is causal.

Convolution : Discrete System

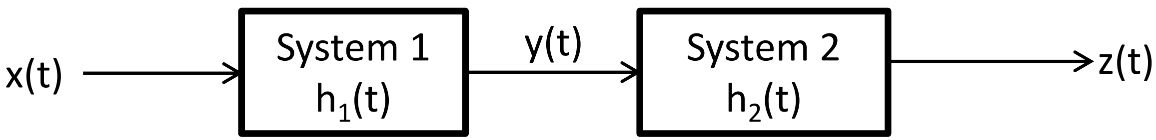
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\ h[n-k]$$

Note that h[n-k] = 0 for n < k when dealing with a *causal* system

e.g., For a perfect amplifier of strength C_1 , h[n-k] = $C_1\,\delta[n-k]$

A convolution chain

An experiment or process can consist of several LTI systems connected in series:



$$z(t) = \int_{-\infty}^{\infty} h_2(t - t'')y(t'')dt''$$

$$= \int_{-\infty}^{\infty} h_2(t - t'') \left[\int_{-\infty}^{\infty} x(t')h_1(t'' - t')dt'\right] dt''$$

e.g., $h_1(t) = K_1 \cos(\omega t + \varphi)\ e^{-K_2 t}$

System 2 is a signal processor: amplifier/filter and time delay

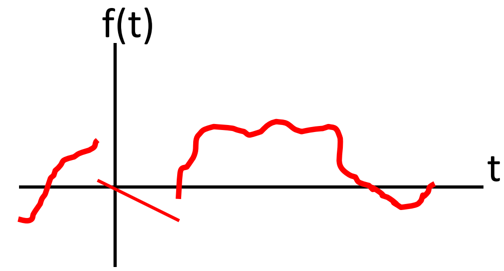
If h_1(t) and h_2(t) are complicated functions, and there is a complicated input signal x(t), it is going to be computationally expensive to find z(t)

$\Rightarrow$ Try Fourier transforms

Fourier Transforms

Consider any piece-wise continuous function f(t).

(or possibly with a finite number of discontinuities)
Monsieur Fourier showed that it can be exactly constructed as a sum of sines and cosines of various amplitudes and frequencies.



This gives us an alternative way to represent the same function:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

where ω = frequency in radians/s = $2\pi f$, and

$$j = \sqrt{-1}$$

F(ω) and f(t) therefore represent exactly the same signal, and contain exactly the same information.
It is analogous to speaking about the same signal but in two different languages.
We say that F(ω) is the frequency domain representation (or Fourier domain representation) of f(t).

For this course, we will assume that the f(t) is always a real function. Note however that F(ω) may be a complex function because of the $e^{-j\omega t}$ term in the integral: $e^{-j\omega t} = \cos(\omega t) - j \sin(\omega t)$

Fourier transforms – Some physical insights

Recall that $e^{-j\omega t}$ = $\cos\omega t - j\sin\omega t$. If *f(t)* is real and is an even function, then

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\, [\cos\omega t - j\sin\omega t\,]dt$$

$$= \int_{-\infty}^{\infty} f(t)\, [\cos\omega t\,]dt + \int_{-\infty}^{\infty} f(t)\, [-j\sin\omega t\,]dt$$

The second integral has an odd integrand, equal to an even function *f(t)* multiplied by a sine wave. Therefore, this integral equals zero.

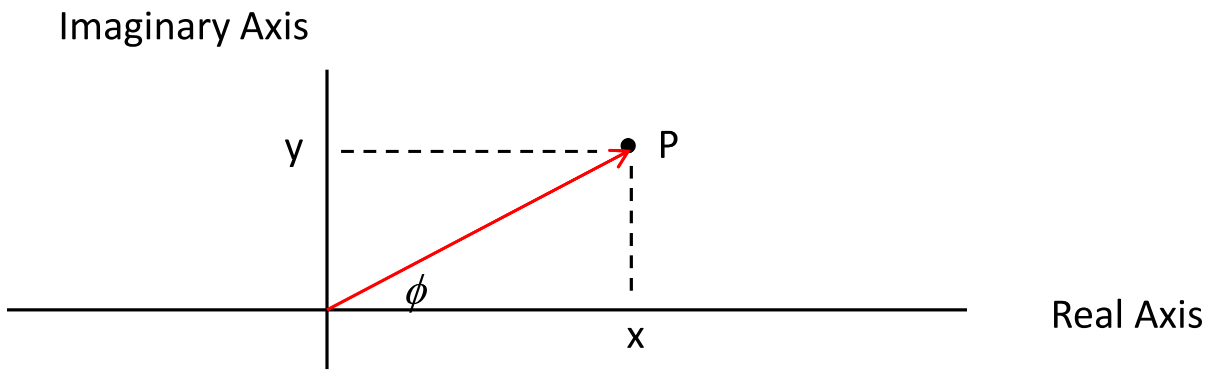
We are left with only the first integral which is real real. So if *f(t)* is real and is even, then F(ω) is real, and F(ω) = F(-ω).

Similarly if *f(t)* is real and odd, then F(ω) is pure imaginary, and F(ω) = F*(-ω) .

If *f(t)* is neither even nor odd, then F(ω) is complex (expressed as real & imaginary components, or as magnitude and phase.)

$\boxed{F(\omega) = F^*(-\omega) \text{ if } f(t) \text{ is real}}$

A problem with complex numbers for signal representation ...



Describe vector as x + jy, or as length R and phase ϕ

where $R = \sqrt{x^2 + y^2}$, and $\phi = tan^{-1}\left(\frac{y}{x}\right)$

Ambiguity: The above diagram shows ϕ = 30° + 2πN, N = ...-2, -2, 0, 1, 2, ...